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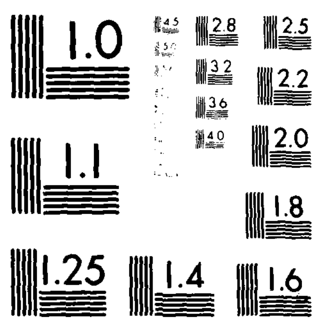

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ACCURATE COORDINATES AND PREDICTED POSITION
OF NAVIGATION SATELLITES NNSS

by

Wladyslaw Goral

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ACCURATE COORDINATES AND PREDICTED POSITION OF NAVIGATION SATELLITES NNSS

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1. Introduction

Satellites of the NNSS navigation system (Navy Navigation Satellite System), or TRANSIT satellites, which in 1967 were made available for commercial flight navigation, have found widespread application in geodetic work. At present, many methods utilize NNSS satellites to determine the coordinates of the observer on the basis of Doppler measurements and the known position of satellites [1, 5, 6].

The position of the SN (navigational satellites) in the system considered is calculated on the basis of data transmitted in 2-minute time intervals from the satellite memory block located on the ship. During this time the SN moves about 900 km in space, which allows determination of the coordinates of the observer from Doppler measurements. The accuracy of determination of coordinates depends, among other things, on geometric conditions of the SN trajectory above the horizon of the observer. The Doppler measurements made during the most favorable flights of the SN above the horizon are used in calculations. For an effective utilization of measurements, and for shortening the computation time, it is necessary to have information about the SN trajectory.

In other words, it is necessary to know their ephemerides. The knowledge of ephemeris is indispensable for turning on the measurement instruments for receiving signals from the SN. This knowledge is also necessary for receivers which automatically receive signals from the satellite as soon as it appears over the horizon. If there are overflights of two SN at approximately the same time, the receiver will accept signals from the satellite which appeared first over the horizon, and will ignore signals of the second satellite. Only an operator can switch the SN signal receiver to a signal with more favorable geometric conditions. On the basis of data transmitted from the SN, one can calculate the ephemeris for any point on the Earth many weeks ahead.

This paper gives practical formulas, tables and graphs which enable fast determination of flight conditions of navigational satellites of the NNSS system, particularly for the terrain of Poland. This is preceded by an example of deciphering information sent by a SN, and calculation of its position in space on the basis of transmitted data.

2. Deciphering of NNSS Message

NNSS navigation satellites have memory blocks. The anticipated elements of the Kepler orbit, the so-called orbital parameters, and information about time is fed into these memory blocks covering a period of about 12 hours. These data are necessary to find the position of the SN in the geocentric system and are transmitted in two-minute intervals on both carrier frequencies (150 MHz and 400 MHz) without disturbing the Doppler measurements. Digital signals sent in 2-minute intervals contain 6103 binary bits, made up of 156 words of 39 bits each and the last 157-th word of 19 bits. These words form 25 groups, each having 6 words of 39 bits (25 groups).

The last group contains 7 words. One group of 6 words is transmitted during $4^{s}600908$. This interval or its multiple is utilized in the short Doppler method. The words with numbers 1, 2, 3 contain information about time, and between the transmission of words 2 and 3 there is the so-called time marker given with the highest possible accuracy, of the order of a few microseconds. The time marker refers to each even TU minute, and is the initial time for the Doppler calculations. Of the remaining 154 words, only one group of 25 words (No. 8, 14, 20, ...152) is utilized in navigation and geodetic practice. The information contained in the remaining words has not been disclosed; it is of a military nature and is not deciphered by civilian receivers [1,2].

The words of interest to us (with numbers 8, 14, ...152) are nine-digit numbers in the decimal system. The first 8 words (with numbers 8, 14, 20, 26, 32, 38, 44, 50) contain information about the so-called orbital parameters. The content of one word refers to one TU expressed in even minutes. During each transmission we are getting ephemeris parameters for a 14-minute interval of time (every 2 minutes), which enables their interpolation for the desired intermediate times. This is done for instance, in short Doppler programs. Each ephemeris word gives the number of 2-minute intervals of time which have elapsed since the last full or half hour of TU time. Once the TU time is known with an accuracy of several minutes or even more than ten minutes from the above mentioned data, we assign a time to the beginning of a given transmission, and times to particular words of the ephemeris. Moreover, in each ephemeris word there is an actual correction to the eccentric anomaly ΔE, semimajor axis $\Delta\alpha$, and partial information about the component normal to the Keplerian orbit $\overline{Z}$.

The deciphering of ephemeris words is rather complex. The first digit of the ephemeris word contains information about

the first time information digit n_t, and information about the signs of corrections ΔE and Δα. The deciphering of the /212 first ephemeris word digit n is given in Table 1 (the first four columns). Now, from the second ephemeris word digit n_t we obtain the number of two-minute intervals which have elapsed since the last full hour (half hour) TU.

n	n_t	Znak ΔE	Znak Δα	n_k	Z_p
0	0	+	+	0	−5
1	0	+	−	1	−4
2	0	−	+	2	−3
3	0	−	−	3	−2
4	1	+	+	4	−1
5	1	+	−	5	+0
6	1	−	+	6	+1
7	1	−	−	7	+2
				8	+3
				9	+4

Table 1.
("Znak = sign)

The next 3 digits give the correction ΔE in units of 0°0001. The digits in positions 6, 7 and 8 of the ephemeris word give the value of the absolute correction Δα in units of 0.01 km. The last (nineth) digit of the ephemeris word contains information about the component $\bar{Z}$. In order to decipher it properly, one has to know accurately the time to which the given ephemeris word refers. The first digit of $\bar{Z}$ is determined on the basis of the value of the 9th digit of the ephemeris word n_k in relation to TU expressed in minutes and divided by 4. Its deciphering is given in the last two columns of Table 1. The second digit of component $\bar{Z}$ is left unchanged, as the ninth digit of the next ephemeris word. It is written next to the value $\bar{Z}_p$ taken from Table 1. Hence, the value of component $\bar{Z}$ is given every 4 minutes. This value consists of two digits and a sign, and it expresses $\bar{Z}$ in units of 0.01 km.

After each consecutive 2-minute transmission, the values of particular ephemeric words move downwards by one line. For instance, the word number 44 goes into the word No. 50, and

the word No. 50 from previous transmission is dropped. The location of word No. 8 from a previous transmission is now taken over by a new value. The parameters pertaining to the given 2-minute transmission are coded in word No. 26 (let this time be called t). Hence, the information contained in word No. 8 refers to the moment $t - 6^m$, and correspondingly $14 \rightarrow t - 4^m$, $20 \rightarrow t - 2^m$, $32 \rightarrow + 2^m$, $38 \rightarrow t + 4^m$, $44 \rightarrow t + 6^m$, and $50 \rightarrow t + 8^m$.

Beginning with number 56 the words contain the actual elements of the Keplerian orbit. On the average, these values undergo changes every 12 hours. The meaning of particular words is as follows:

- the word No. 56 contins value of the time interval which has elapsed from midnight GMT to the time of the first transit of the satellite through perigee after the last introduction of data into the SN memory block of (t_p);
- the word No. 62 contains the value of average motion η;
- the word No. 68 contains the value of the argument of perigee ω at the time t_p;
- the word No. 74 contains the absolute value of the change of argument of perigee $|\omega|$;
- the word No. 80 contains the value of excentricity of the orbit e;
- the word No. 86 gives value of the semimajor axis α;
- word No. 92 gives value of the longitude of the ascending node Ω at time t_p;
- word No. 98 gives value of the change of longitude of the mode $\overset{\circ}{\Omega}$;
- Word No. 104 gives value of the cosine of the inclination of the orbit, cos i;
- Word No. 110 gives value of Greenwich sidreal (star) time S at the moment t_p;
- word No. 116 gives value of the number of the navigation satellite N;

- word No. 122 contains the value t_w, of the time of introducing data into the SN memory block and the number of current days counted from the beginning of the given year;
- word No. 128 gives value of the sine of the inclination of the orbit, sin i;
- word No. 134 contains value of Δf - deviation from the frequency 400 MHz transmitted by the SN transmitter; this value is expressed in units of 80×10^{-6};
- words with numbers 140, 146 and 152 usually contain zeros.

The first decimal number of the word No. 56 always has the value of 0 or 4, and is deciphered as +0, or +1 in the case of 4.

The first digit of words with numbers 62, 68, ...134 is always 8 or 9; it gives the sign of a given value, the sign plus (+) is denoted by 8 and the sign minus (-) by 9. The last, i.e., the 9th figure of words 62, 68 ...135 is always zero and may be omitted.

The elements of the orbit, defined by fixed parameters contained in words No. 56, ..., 128, extrapolate the orbit of a given SN. The elements of the orbit actually undergo constant changes because of the action of various perturbing forces, caused mainly by the gravitational field of the Earth and friction of upper atmosphere. The effect of this part of the perturbations which can be predicted is given in the parameters of the ephemeris.

Table 2 gives an example of a printout by a navigational receiver of a message transmitted from SN-18, and gives its detailed deciphering.

3. Position of SN in Space

On the basis of data contained in Table 2, we shall calculate

the Greenwich geocentric coordinates of SN-18. The numerical example refers to the time $t = 13^h00$ TU on 17 December 1975. Using formulas for Keplerian motion, we calculate value of the mean anomaly M.

$$M = n(t - t_p) = n(780^m - 446^m0124) = 44°61514 \qquad (1)$$

Taking into consideration this value of M in solving the Kepler equation $E_o = M + e \sin E_o$ (by the method of successive approximations or by Newton's method) we obtain the value of the excentric anomaly $E_o = 44°91811$. Taking from Table 2 the

Table 2

Word No.	Print-out	Deciphered parameters of ephemeris			
		t	ΔE	Δp [km]	z [km]
8	42 018 269 0	12 (12h54m	+0°0018	+2,69	
14	43 033 250 5	13 (12 56	+0 0023	+2,50	+0,04
20	44 046 236 4	14 (12 58	+0 0046	+2,24	
26	00 051 193 5	0 (13 00	+0,0051	+1,93	+0,00
32	01 063 158 8	1 (13 02	+0,0062	+1,58	
38	02 065 121 6	2 (13 04	+0 0065	+1,21	+0,11
44	03 065 086 7	3 (13 06	+0 0066	+0,85	
50	04 061 063 6	4 (13 08	+0 0061	+0,51	+0,14
		Orbital elements (permanent part of message			
56	8 44601240	$t_p = 446^m0124 = 7^h26^m0124$ TU			
62	8 36723620	$n = 3°3672362$ min^{-1} (1)			
68	8 34701100	$\omega = 347°0110$			
74	8 00200780	$\lvert\dot{\omega}\rvert = 0°0020078$ min^{-1} (2)			
80	8 00074090	$e = 0,007409$			
86	8 07461850	$a = 7461,85$ km			
92	8 27602540	$\Omega = 276°0254$			
98	8 00000040	$\dot{\Omega} = 0°0000004$ min^{-1}			
104	8 00003280	$\cos i = 0,000328$			
110	8 19680970	$S = 196°8097$			
116	8 03018000	$N = 3018$ numer SN			
122	8 18803510	$t_w = 188 \times 2^m = 376^m = 6^h16^m$TU w 351 dniu 1975 r., to jest 17go, XII 1975 r.			
128	8 10000000	$\sin i = 1,000000$			
134	8 00175000	$\Delta f_0 = 80,0175 \cdot 10^{-6} \cdot 400$ MHz $= 32,007$ kHz			

Note: 1) Transmission from the SN gives only the fractional part of the mean motion. This value has to be always preceded by the number 3 denoting degrees;

2) For SN TRANSIT the rate of change of the argument of perigee (the abscissa) always has a negative value ($\dot{\omega}<0$, since $i \simeq 90°$).
The deciphering gives nominal accuracies and units in which the transmitted values are expressed. Values of ω, Ω, S refer to time t_p.

correction $\Delta E = 0°0051$ and adding it to E_o we obtain:

$$E = E_0 + \Delta E = 44°92321 \qquad (2)$$

The value of semimajor axis is

$$a = a_0 + \Delta a = 7463{,}78 \text{ km} \tag{3}$$

In turn, we calculate the radius r from the formula

$$r = a(1 - e\cos E) = 7424{,}2024 \text{ km} \tag{4}$$

In practice, we calculate first the orbital coordinates of the SN (the system origin is in the center of Earth, the axis $\overline{X}$ is directed to perigee, the axis $\overline{y}$ lies in the plane of the orbit and is directed in the direction of motion of the SN, and the axis $\overline{Z}$ - normal to the plane of orbit - compliments the system to make it clockwise). Calculations are performed with the aid of formulas

$$\begin{aligned} \bar{x} &= a(\cos E - e) = 5228{,}862 \text{ km} \\ \bar{y} &= a\sqrt{1-e^2}\sin E = 5270{,}464 \text{ km} \end{aligned} \tag{5}$$

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$\bar{z}$ = 0.08 km - as a parameter of ephemeris taken from Table 2

Check: $r = (\bar{x}^2 + \bar{y}^2 + \bar{z}^2)^{1/2}$ = 7424.202 km, agreeing with (4)

From the rectangular orbital system we can transform into the rectangular equatorial (equinocturnal) or Greenwich system by means of vectors $\overline{P}$, $\overline{Q}$, $\overline{R}$ [4], which are functions of the following orbital elements: Ω, i, ω. Since in practical satellite applications we are interested in direct transformation into the Greenwich system, instead of angle Ω we substitute the angle $\Omega - S = \lambda$. For a given moment t we obtain

$$\begin{aligned} \Omega - S &= \Omega(t_p) - S(t_p) + (\Omega - \dot{S})(t - t_p) = -4^\circ 50967 \\ \omega &= \omega(t_p) + \dot{\omega}(t - t_p) = 346^\circ 34042 \end{aligned} \tag{6}$$

Values of the quantities $\Omega(t_p)$, $\omega(t_p)$, $\mathring{\Omega}$, $|\dot{\omega}|$, sin i and cos i are obtained from Table 2. $\mathring{S}$ - 0°250684476 min^{-1} is the value of the average rate of rotational motion of the Earth.

The components of vectors $\overline{P}$, $\overline{Q}$, $\overline{R}$ are calculated from equations

$$
\begin{aligned}
P_x &= \cos\omega \cos(\Omega - S) - \sin\omega \sin(\Omega - S)\cos i = 0{,}9687015 \\
P_y &= \cos\omega \sin(\Omega - S) - \sin\omega \cos(\Omega - S)\cos i = -0{,}0764807 \\
P_z &= \sin\omega \sin i = -0{,}2361527 \\
Q_x &= -\sin\omega \cos(\Omega - S) - \cos\omega \sin(\Omega - S)\cos i = \\
&= 0{,}2354466 \\
Q_y &= -\sin\omega \sin(\Omega - S) + \cos\omega \cos(\Omega - S)\cos i = \\
&= -0{,}0182503 \\
Q_z &= \cos\omega \sin i = 0{,}9717159 \\
R_x &= \sin(\Omega - S)\sin i = -0{,}0786274 \\
R_y &= -\cos(\Omega - S)\sin i = 0{,}9969041 \\
R_z &= \cos i = 0{,}0003280
\end{aligned}
\qquad (7)
$$

Check: $|\bar{P}| = 1$, $|\bar{Q}| = 1$, $|\bar{R}| = 1$, $\bar{P}\cdot\bar{Q} = \bar{P}\cdot\bar{R} = \bar{Q}\cdot\bar{R} = 0$.

On the basis of (5) and (7) we obtain the desired SN rectangular Greenwich coordinates from equations

$$
\begin{aligned}
x &= \bar{x}P_x + \bar{y}Q_x + \bar{z}R_x = 6306{,}113 \text{ km} \\
y &= \bar{x}P_y + \bar{y}Q_y + \bar{z}R_y = -496{,}174 \text{ km} \\
z &= \bar{x}P_z + \bar{y}Q_z + \bar{z}R_z = 3886{,}584 \text{ km}
\end{aligned}
\qquad (8)
$$

Let r denote the geocentric radius of the SN, λ and ϕ geocentric longitude and latitude - the coordinates of the sub-satellite point. When we take into consideration the following formulas

$$
\begin{aligned}
x &= r\cos\varphi\cos\lambda \\
y &= r\cos\varphi\sin\lambda \\
z &= r\sin\varphi
\end{aligned}
\qquad (9)
$$

and values from (8), we obtain γ = 7424.202 km, $\lambda = -4^\circ 4988$, $\phi = 31^\circ 5674$.

Knowing the position of the SN in the rectangular geocentric system and the position of an observer in the same system, or in a geodetic system on a given reference ellipsoid, one can calculate the horizontal coordinates of the satellite. We are particularly interested in the horizontal height (altitude) h. This parameter is necessary when calculating corrections because of tropospheric refraction.

4. Predicted Position of NNSS Satellites

In predicting flight conditions of a particular SN over

a given location on the surface of the Earth, the accuracy with which one can extrapolate changing elements of the orbit over larger time intervals is very important.

Table 3 gives transmitted orbital elements of satellites No.s 13, 14, 12, 19 and 20. The elements of SN-18 were given in Table 2. All these data refer to satellites operating in December 1975.

Table 4 lists transmitted orbital elements of satellites No.s 20, 14, 13, 12 and 19 observed in January 1976.

In Tables 3 and 4, the parameters n, $|\mathring{\omega}|$, and $\mathring{\Omega}$ are expressed in units of 1 degree/minute angular velocity. The semimajor axis α is in km, and the remaining parameters and orbital elements are given in Tables. The second part of Table 4

Table 3

1 - Satellite	13	14	12	19	20
Data	11 XII	14 XII	17 XII	17 XII	17 XII
t_p	566m9556	676m8215	235m5248	317m1386	1247m7382
n	3,3663232	3,3728142	3,3815153	3,3655759	3,4104949
ω	177°1699	178°5254	264°8926	85°4456	177°2318
$\lvert\dot{\omega}\rvert$	0,0019914	0,0019973	0,0020265	0,0019373	0,0020342
e	0,001551	0,004795	0,000792	0,018974	0°016857
a	7463,19	7453,62	7440,83	7464,30	7398,62
Ω	314°2207	326°4641	22°9891	251°5600	121°5799
$\dot{\Omega}$	—0,0000248	—0,0000509	+0,0000139	+0,0000091	0,0000125
cos i	0,006553	0,013098	—0,003900	—0,002004	—0,002609
S	224°1714	248°7561	144°0438	164°5031	37°7900
sin i	0,999979	0,999914	0,999992	0,999998	0,999997

1- satellite

gives elements ω_c, Ω_c calculated from equations

$$\begin{aligned} \omega_c &= \omega + (n - |\dot{\omega}|)\Delta t \\ \Omega_c &= \Omega + \dot{\Omega}\Delta t \end{aligned} \tag{10}$$

on the basis of initial data for ω, Ω, n, $|\mathring{\omega}|$, $\mathring{\Omega}$ taken from Table 3. The calculations were made for appropriate times t_p of particular SN - data in Table 4. Hence $\Delta t = t_{p4} - t_{p3}$

Table 4

Satellite —	20	16	13	12	19
Date	23 I	23 I	23 I	26 I	26 I
t_p	324m0824	455m0701	516m9192	661m3586	1012m9570
n	3,4104472	3,7737977	3,3663090	3,3814999	3,3656753
ω	74°8025	72°6887	94°3499	129°0120	332°3382
$\lvert\dot{\omega}\rvert$	0,0019823	0,0019707	0,0019726	0,0020026	0,0020171
e	0,017976	0,006090	0,003260	0,002766	0,017347
a	7398,68	7453,64	7468,22	7440,85	7464,16
Ω	122°1480	323°2304	312°7270	23°8840	252°0197
$\dot{\Omega}$	0,0000094	—0,0000512	—0,0000244	—0,0000173	0,0000620
$\cos i$	—0,002680	0,013027	0,006487	0,003890	—0,001942
S	202°9136	235°5516	240°5501	287°7111	18°3601
$\sin i$	0,999997	0,999915	0,999978	0,999992	0,999998
dt	36d517m1442	42d1210m2566	39d1383m9636	40d415m0868	40d695m8184
S_0	202°9182	235°5512	240°5498	287°7107	18°3598
ω_c	74°5001	72°0514	94°1565	128°7261	331°5101
$\omega - \omega_c$	0°3024	0°6373	0°1934	0°2859	0°8277
Ω_c	122°2343	323°3236	312°7936	23°7955	252°0905
$\Omega - \Omega_c$	—0°0862	—0°0933	—0°0666	0°0885	—0°0708

is the time interval which elapsed between the transit of the SN over perigee given in Table 3 and the same moment given in Table 4. Table 4 also gives the differences $\omega-\omega_c$ $\Omega-\Omega_c$ between elements ω, Ω taken from Table 4 and those calculated using the values ω_c, Ω_c.

Comparing the orbital elements of particular SN from Table 3 and those after 5-6 weeks (Table 4), we see that the accurate calculation of predicted position, or ephemeris, is affected most significantly only by the difference $\omega-\omega_c$, which in some cases is smaller than 1°. The remaining elements remain the same, or change to a small extent. The sidereal (star) time S_c, calculated from equation

$$S_c = S + \dot{S}\Delta t \tag{11}$$

differs very little from the corresponding values of S in Table 4, as a result of the simplified form of equation (11).

Since orbits of TRANSIT system satellites are nearly circular and polar ($e \approx 0$, and $i \approx 90°$), in calculations of the ephemeris we assume that $e = 0$, $i = 90°$ (the circular and polar orbit). Moreover, we assume that the Earth is a sphere with a mean radius $R = 6365$ km. These simplifying assumptions facilitate the calculations considerably, and still enable us to obtain sufficient accuracy when computing the ephemerides.

On the basis of transmitted orbital elements we shall first calculate the longitude of the ascending node λ_w, that is the geographic longitude of a point on the equator at which the SN passes from the Southern to the Northern hemisphere, /214
and also the transit time through this point

$$t_w = (t_p) - S(t_p) + (\dot{\Omega} - \dot{S})(t_w - t_p) \tag{12}$$

The SN in circular orbit moves with a uniform angular rate n,

and its longitude along the orbit u at the time t is calculated from equation

$$u(t) = \omega(t_p) + (n - |\dot{\omega}|)(t - t_p) \tag{13}$$

Let the longitude of the node λ_W be its longitude at the first transit of the SN through the equator from South to North at the time t_p transmitted in the message. Then $u(t_W) = 360°$. Substituting this value in (13) we get

$$t_w = t_p + \frac{360° - \omega(t_p)}{n - |\dot{\omega}|} \tag{14}$$

Let T be the time period required by the SN to fly around the Earth. It is expressed by equation

$$T = \frac{360°}{n - |\dot{\omega}|} \tag{15}$$

It is called the nodal period. After one orbit, the longitude of the initial node will change by $\Delta\lambda_\omega$, and

$$\Delta\lambda_w = (\dot{\Omega} - \dot{S})T \tag{16}$$

therefore after k orbits we have

$$\begin{aligned} \lambda_w^k &= \lambda_w^0 + k\Delta\lambda_w \\ t_w^k &= t_w^0 + kT \end{aligned} \tag{17}$$

where:

λ_W^o, t_W^o - initial value of the node, and the transit time of the SN through this point, calculated from equations (12) and (14),

On the basis of given values of parameters λ_W, t_W we calculate spherical coordinates of subsatellite points (their geocentric longitude and latitude) at any moment t. The longitude is calculated from equation

$$\lambda(t) = \lambda_w + (\dot{\Omega} - \dot{S})(t - t_w) \tag{18}$$

We assume that the geographic longitude is calculated positive to the East of the Greenwich meridian ($0° < \lambda_E < 180°$), and negative to the West of the Greenwich meridian ($-180° < \lambda_W < 0°$). Hence, after the SN passes through the North

pole from the Eastern to Wesern hemisphere we subtract 180° from the value calculated from equation (18). Conversely, we add 180° after passing the pole from the Western to Eastern hemisphere.

When calculating the latitude ϕ we have the following three cases:

a) $\phi = u$, the SN moves from the equator to the North Pole

$$0^\circ \leqslant u \leqslant 90^\circ \tag{19}$$

b) $\phi = 180^\circ - u$, the SN moves from the North Pole to the South Pole

$$\text{for} \quad \text{dla } 90^\circ \leqslant u \leqslant 270^\circ \tag{20}$$

c) $\phi = -360^\circ + u$, the SN moves from the South Pole to the equator

$$\text{for} \quad \text{dla } 270^\circ \leqslant u \leqslant 360^\circ \tag{21}$$

where:

$$u = (n - |\dot{\omega}|)(t - t_w) \tag{22}$$

Let the coordinates of an observer (coordinates of the point for which we want to calculate the ephemeris) be λ_o, ϕ_o. First, we calculate the time of apogee t_k. For an SN in polar orbit, the SN in at apogee is approximately at the latitude of the observer. For an observer in the Northern hemisphere two cases can arise, i.e., case a)for which equation (19) applies, or case b) for which equation (20) is relevant.

For apogee, the longitude of the satellite in orbit is

$$\begin{aligned} &\text{a)}\ u = \varphi_0 \\ &\text{b)}\ u = 180^\circ - \varphi_0 \end{aligned} \tag{23}$$

Knowing u we calculate the time of apogee t_k by substituting (23) into (22)

$$\text{a) } t_k^a = t_w + \frac{\varphi_0}{n - |\dot{\omega}|}$$

$$\text{b) } t_k^b = t_w + \frac{180° - \varphi_0}{n - |\dot{\omega}|} \qquad (24)$$

The longitude of the subsatellite point at the apogee of the SN is expressed by the equation

$$\lambda_k = \lambda_w + (\dot{\Omega} - \dot{S})(t_k - t_w) \qquad (25)$$

When $\lambda_k > \lambda_o$, the SN moves in the Eastern side of sky,
When $\lambda_k = \lambda_o$, the SN passes through zenith,
When $\lambda_k < \lambda_o$, the SN moves in the Western side of sky.

On the basis of equations (18) and (24) we can calculate the longitude of the ascending node for an NSSZ passing through zenith. The condition to be fulfilled is

$$\lambda_0 = \lambda_w + (\dot{\Omega} - \dot{S})(t_k - t_w) \qquad (26)$$

For the cases a) and b) we obtain, respectively

$$\text{a) } \lambda_w^a = \lambda_0 - (\dot{\Omega} - \dot{S})\frac{\varphi_0}{n - |\dot{\omega}|} \qquad (27)$$

$$\text{b) } \lambda_w^b = \lambda_0 - (\dot{\Omega} - \dot{S})\frac{180° - \varphi_0}{n - |\dot{\omega}|} \pm 180° \qquad (28)$$

The sign (-) refers to the case in which $\lambda_o > 0°$ (the observer is in the Eastern hemisphere), and the sign (+) applies to the situation when $\lambda_o < 0°$ (the observer is in the Western hemisphere).

In turn, we calculate the maximum interval of time of the SN stays over the horizon, which takes place when a given satellite passes through zenith (apogee at zenith).

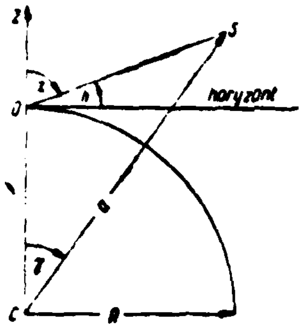

Figure 1

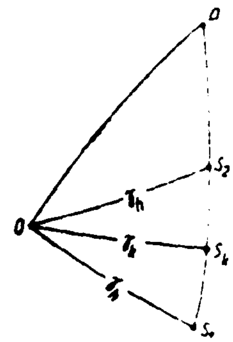

Figure 2

Let Z denote the zenith distance of the satellite, and γ the geocentric angle between the direction to zenith and direction to the SN (Figure 1). From Figure 1 we get the relation

$$\frac{a}{\sin z} = \frac{R}{\sin(z-\gamma)} \qquad (29)$$

Solving the equation with respect to Z we get

$$\operatorname{tg} z = \frac{\sin\gamma}{\cos\gamma - \frac{R}{a}} \qquad (30)$$

where:

R - average radius of Earth

α - distance of SN from the center of Earth

For an SN which is on the horizon (Z = 90°) we have according to (29)

$$\cos\gamma_h = \frac{R}{a} \qquad (31)$$

As an example, assuming R = 6365 km and α = 7440 km (the value of the semijajor axis for SN-12, which is equal approximately to the average value of semimajor axis NSS considered in Tables 2 and 3) we get

$$\gamma_h \approx 31°18 \qquad (32)$$

If the SN reaches apogee at zenith, then the maximum stay time Δt_{max} of a given SN on the horizon is approximately

$$\Delta t_{max} = \frac{2\gamma_h}{n - |\dot{\omega}|} \tag{33}$$

since $\Delta u = 2\gamma_h$ and we have agreement with (22). /215
From the data for SN-12 we get $\Delta t_{max} = 18^m45$.

The time interval which elapses between the rising (appearance) of a given SN and its transit through zenith is $1/2\ \Delta t_{max}$.

The above values of γ_h and Δt calculated for SN-12, and the conclusions, are also valid for the remaining SN of the TRANSIT system, since the values of orbital elements as such a, e, i, n are very close.

The time interval sutiable for Doppler measurements is shorter than the maximum staying time of the SN above the horizon, since in computations based on integrated Doppler observations no near-horizon and near-zenith measurements are used. The maximum number of integrated Doppler type measurements - taken during one flight of an SN over a 2-minute time interval does not exceed 8-9.

In turn, we shall discuss conditions of visibility of an SN depending on the geographic latitude of the observer. According to (16) during one pass of an SN, its orbit will move in the plane of equator to the West. For example, for SN-12 we get

$$\Delta\lambda_w = -26^\circ70 \tag{34}$$

Taking this into consideration and the value (32) we see that for near-equatorial regions, the SN passes over the horizon of selected point only during 2-3 consecutive passes - once on passing through the equator from the Southern to Northern

hemisphere (ascending orbit), and the second time half a day later on passing from the Northern to Southern hemisphere (descending orbit). In near-polar regions the SN will be over the horizon during each flight. The number of flights of the SN per day, useful for geodetic and navigational work, increases with geographic latitude of the observer from 4-6 in areas near the equator to 12-14 in near-polar regions.

For an observer at a given latitude ϕ_o the SN will be over the horizon only when the longitude of the ascending node λ_w is close to λ_w^a, λ_w^b - see equations (27) and (28). The difference should not be greater than the value of $\Delta\lambda_m$ from equation

$$\sin \Delta \lambda_m = \frac{\sin \gamma_h}{\cos \varphi_o} \tag{35}$$

This equation follows from the spherical triangle OPS (Figure 2). In the limiting case for a given SN reaching apogee on the horizon we have $\gamma = \gamma_h$, $\Delta\lambda = \Delta\lambda_m$. The internal angle at the apexes is a right angle.

Table 5

φ_o	$\Delta\lambda_m$	φ_o	$\Delta\lambda_m$
0°	31°2	50°	53°7
10°	31°7	55°	64°5
20°	33°4	58°	77°7
30°	36°7	58°5	82°0
40°	42°5	58°82	90°0

Table 5 gives values of $\Delta\lambda_m$ calculated according to (35) after accepting the value (32). It follows also from equation (35) that for $\Delta\lambda_m = 90°$ we have $\gamma_h = 90° - \phi_o$, hence for points satisfying the condition

$$|\varphi_o| > 90° - \gamma_h \tag{36}$$

the SN will be over the horizon of a given observer during each flight.

From triangle OPS we also get the relation

$$\operatorname{tg} \varphi_s^k = \frac{\operatorname{tg} \varphi_0}{\cos \Delta \lambda^k} \tag{37}$$

where:

$$\Delta \lambda^k = \lambda_s^k - \lambda_0; \tag{37a}$$

ϕ_s^k, λ_s^k - geographic latitude and longitude of the subsatellite point at the time of apogee of a given SN.

If we take into consideration the effect of the rotational motion of the Earth, then the formula for geographic latitude of the subsatellite point has the form

$$\operatorname{tg} \varphi_s^k = \frac{\operatorname{tg} \varphi_0}{\cos \Delta \lambda^k} - \frac{(\dot{\Omega} - \dot{S})}{n - |\dot{\omega}|} \operatorname{tg} \Delta \lambda^k \tag{38}$$

For an SN culminating rather high over the horizon ($\cos \Delta\lambda \approx 1$) the following approximate realtion applies, in conformity with (37) or (38)

$$\varphi_s \approx \varphi_0 \tag{39}$$

For a SN TRANSIT satellite at the time of apogee the geographic latitude of the subsatellite point is approximately equal to the latitude of the observer.

For example, for $\phi_0 = 52°$ and $\Delta\lambda^k = 20°$ the SN reaches apogee at the zenith distance $z \approx 60°$, at $\phi_s = 53°71$ according to (37) or at $\phi_s = 54°24$ according to (38). The difference $\phi_s - \phi_0 \approx 2°24$ is covered by the SN during 40^s. This difference will be smaller for SN reaching apogee higher above the horizon and larger at lower altitudes. In computations of the predicted ephemeris of a polar SN we assume that $\varphi_s = \phi_0$ - see equations (23), (24) and (27).

As an example, let us consider the coordinates of a point

in the area of Poland($\lambda = 20°$, $\phi = 52°$). For SN-12, according to (35) we obtain $\Delta\lambda_m = 57°24$. Hence, from (27) and (28) we get

$$\lambda_w^a = 33°86; \quad \lambda_w^b = -150°50 \tag{40}$$

The time intervals between the crossing of the equator by a given SN and apogee are, in conformity with (24)

$$t_k^a - t_w = 15^m4; \quad t_k^b - t_w = 37^m9 \tag{41}$$

Knowing the moment of passage of the SN through the node we calculate the time of apogee, on the basis of (41). The SN will fly over the horizon of point P if the following conditions are fulfilled for the longitude of the ascending node:

a) $-33°4 \leq \lambda_w \leq 81°1$ SN moves in Soutn-North direction

(42)

b) $152°2E \leq \lambda_w \leq 266°7E$ SN moves in North-South direction

Taking the values of (34) and (42) we see that a given SN flies over the horizon of point P 4-5 times a day in an ascending orbit and similarly on a descending orbit. Knowing the apogee point t_k and λ_w, we calculate from (18) the longitude of the subsatellite point at the time of apogee λ^k and (see Figure 2)

$$\sin\gamma_k = \cos\varphi_0 \sin\Delta\lambda^k \tag{43}$$

Knowing the geocentric angle γ_k, we calculate from equation (30) the zenith distance of the SN at the time of apogee. At the ascending or descending time of the SN we have $\gamma = \gamma_h$, hence from the cosine formula for spherical triangle OS_kS_1 (Figure 2) we have

$$\cos S_1S_k = \frac{\cos\gamma_h}{\cos\gamma_k} \tag{44}$$

Since $S, S_k = (n-|\mathring{\omega}|\Delta t$, the time interval which elapses between the ascension and apogee, or apogee and descent of the SN is expressed by the equation

$$\Delta t = \frac{S_1 S_k}{n-|\mathring{\omega}|} \tag{45}$$

Equations (44) and (45) are approximate, since they were deduced on the assumption that $\Delta\lambda^{k} = \Delta\lambda^{East} = \Delta\lambda^{West}$, the angle $\Delta\lambda$ does not change during the motion of the SN over the horizon and is equal to its value at apogee, or the average value during the flight of the SN over the horizon.

On the basis of these formulas we can calculate the ephemeris of an SN for any point on the surface of the Earth during the day. On the next day, the conditions of flight over the horizon of a given point repeat themselves approximately after 12-13 passes around the Earth. Hence, we add the values of 12 $\Delta\lambda_w$, 12 T or 13 $\Delta\lambda_w$, 13 T to the corresponding values of ascending λ_w, t_w at which SN appears on the horizon.

The discussed equations can be used to prepare computed ephemerides for SN on a computer. In [3] there is a published program in Fortran language to calculate the ephemerides for an SN at medium latitudes and in areas near the equator omitting the polar regions.

The data necessary to predict the ephemerides of SN TRANSIT /216 satellites are also provided by the Defense Mapping Agency, Hydrographic Center, Washington, D.C. 20390 [3]. These give the values of λ_w, t_w, $\Delta\lambda_w$, T. Table 6 gives the same parameters calculated for 6 navigation satellites for December 17, 1975. They were calculated on the basis of orbital elements given in Table 3 and for SN-18 in Table 2. The parameters and orbital elements given in Tables 2 and 3 were taken from Doppler measurements of a TRANSIT SN made by the Higher Naval Academy in Gdynia.

Table 6

Nr SN	t_w [TU]	λ_w	T	$\Delta\lambda_w$
12	$263^{m}6673$	$-128^{\circ}1092$	$106^{m}5278$	$-26^{\circ}703$
13	$46^{m}4708$	$-142^{\circ}5858$	$107^{m}0049$	$-26^{\circ}827$
14	$100^{m}5864$	$-144^{\circ}1625$	$106^{m}7990$	$-26^{\circ}778$
18	$449^{m}8698$	$+78^{\circ}2483$	$106^{m}9764$	$-26^{\circ}817$
19	$398^{m}7628$	$+66^{\circ}5957$	$107^{m}0269$	$-26^{\circ}829$
20	$1301^{m}3601$	$+70^{\circ}3483$	$105^{m}6195$	$-26^{\circ}476$

5. Graphic Method of Predicting SN Flights

Predictions for the flight conditions of SN over a given point on the surface of Earth can be considerably accelerated by the use of a graphic method. Figure 3 shows a contour map in Mercator projection with the point P(λ= 20°, ϕ = 52° - central Poland). At this point a net is drawn for equal altitudes (almucantars) for the following horizontal elevations 0°(horizon), 7°5, 30° and 60°, and of azimuths (verticals) marked in intervals of 30° in the range 0°-360°. This net was calculated for a SN circling at a distance of α = 7440 km from the center of Earth. On a transparent paper and in the same projection, the path of several consecutive SN passes (for ascending and descending orbits) is drawn. In this case it is sufficient to know only the values of λ_w, t_w, T from one (any) flight on a day to be able to determine visibility conditions of a SN at any point on the surface of Earth. As an illustration, 4 consecutive flights of SN-12 (in ascending and descending orbits) visible over the horizon in Poland on 17 December 1975 were drawn in Figure 3. The graphs were made on the basis of data given in Tables 3 and 6. Assigning No. 1 to an orbit whose ascension is defined by the parameters t_w = 263^m6673(TU), λ_w = -128°109, the following passes will be visible at point P: 0, 1, 2, 3 (descending orbit) and 7, 8, 9, 10 (ascending branch of the orbit, marked in Figure 3 by dots every 2^m).

In order to rapidly determine the moment of time at which the SN will be over a defined point on the surface of the Earth (for determination of the time of apogee t_k), the position of the SN was marked every 2^m, counting from zero at the ascending node. Thus, for example, for pass No. 2 over the horizon at point P we get $t_k^2 = t_w + T + 38^m \approx 6^h48^m2$. For pass

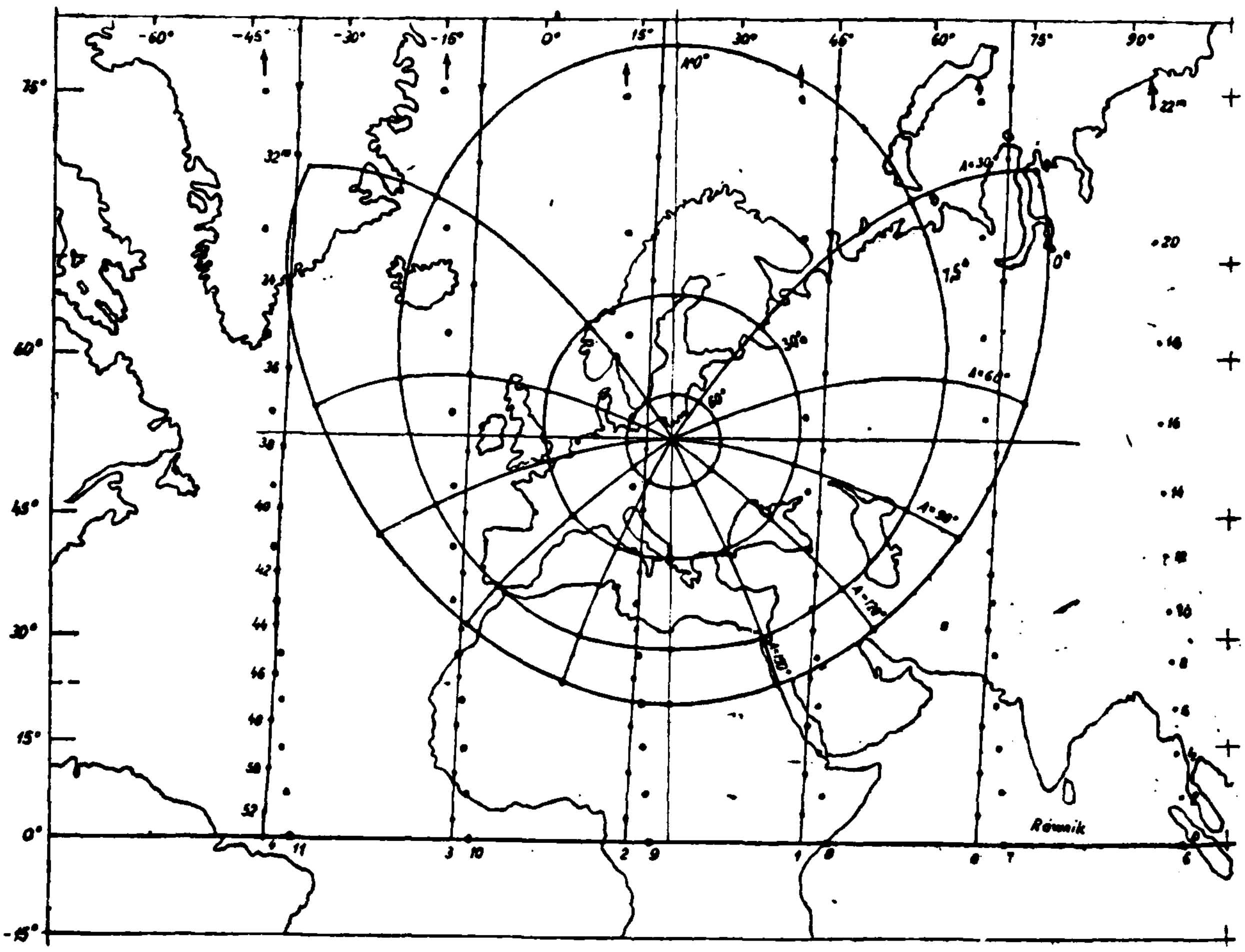

Figure 3

(Rownik = equator)

No. 8 we obtain $t_k^8 = t_w + 7T + 16^m \approx 17^h 5^m 4$. During passes No.s 0, 1 the satellite moves on the Eastern side of the sky, and during passes No.s 2, 3 - on the Western side. In these 4 cases the ascension of the SN is on the Northern side of the horizon, and the descent - on the Southern side. In turn, during passes No.s 7, 8, 9, 10 the ascension is on the Southern side, and the descent - on the Northern side of the horizon. The passes No.s 7, 8 take place on the Eastern side, and No.s 9, 10 - on the Western side of the horizon.

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